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Seriation and Joint Probability: Unifying Archaeological Dates in the Central Mediterranean, ca. 4th-1st c. BCE

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Abstract

Probability theory has been applied to archaeological chronology since the advent of radiocarbon dating, but key chronological indicators, such as artifact typologies, continue to be dated in descriptive terms (e.g., "in the later 3rd c. BCE"). Moreover, the informational inter-dependency for how contexts and assemblages affect others' dating can become opaque. This paper illustrates the use of two new `R` packages, `lakhesis` and `eratosthenes`, to construct a wholly formal method for archaeological dating, focusing on the central Mediterranean in the last four centuries BCE. Contextual seriation is used to compare and sequence disparate contexts alongside stratigraphic relationships, to define a joint probability density function. A case study of tombs, stratigraphic units, shipwrecks, and other depositional events is then implemented for the last four centuries BCE, illustrating how the formal estimation of events is to be achieved. The resulting marginal densities can then be checked against conventional dates, not just to revise typological distinctions and revisit comparisons where discrepancies emerge, but above all to obtain a precise, probabilistic estimation of the occurrence of all archaeological events.

Keywords: chronology; order statistics; seriation; radiocarbon dating; shipwrecks; coinage

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1 Introduction

Combining sources of chronological information in a formal (i.e., mathematical) way has been a clear objective in archaeology (Levy 2025; Noukpoape *et al.* 2025). The development of probabilistic methods in chronology has largely centered on radiocarbon dates, dendrochronology, and other scientific methods, in combination with stratigraphic relationships or other historical markers (Aitken 1990; Buck *et al.* 1991; Buck *et al.* 1996; Bronk Ramsey 1995, 2009; Nicholls and Jones 2002; Dee *et al.* 2013; Taylor and Bar-Yosef 2014; Steier and Rom 2000; Buck and Juárez 2025). In contrast, artifact dates—at least in the Mediterranean—have remained largely within the domain of more qualitative or ambiguous periodizations. Artifacts though are key sources of chronological information, whether intrinsically (such as a datable coin) or extrinsically (in comparison with other finds), and, as this paper shows, their chronological information can be directly incorporated into the formal estimation of events via seriation. The main virtue of this approach is that it allows for the determination of dates in a consistent fashion, using seriations that are based directly on associations in material culture and measurable by optimality criteria. Representing all events as a probability density function (p.d.f.) not only allows for expedient revision of dates, unifying and synchronizing the relationships between events, but further enables the use of more advanced statistical models that can resample from such densities.

To those ends, this paper introduces two new R packages (R Core Team 2024). The first package, `lakhesis`, offers a visual, interactive interface in which to perform seriations (O'Brien and Lyman 1999; Ihm 2005; Hahsler *et al.* 2008), using correspondence analysis (CA) and Procrustes-fit CA (Collins-Elliott 2026b). Here, contextual seriation (i.e., seriation using a binary matrix of the presence/absence of a find-type in a context) is preferred over frequency seriation, since frequency or count data is not available for every context, and may not be comparable within the same data matrix (i.e., if one project has reported minimum number of individuals for ceramics, and another sherds); or, there may be no quantitative information given at all. Contextual seriation in other words provides a versatile "lowest common denominator" in terms of methodology across datasets.

The second R package, `eratosthenes` (Collins-Elliott 2026a), provides tools for estimating the dates of all archaeological events as marginal p.d.f., from the input of sequences, finds, and absolute constraints that comprise a joint, conditional p.d.f. This package does not provide functionality for the calibration of radiocarbon dates, given preexisting software (in this paper, `Bchron` by Haslett and Parnell (2008) is used). Rather, calibrated dates are used as input alongside any other absolute markers, which are classified as either *termini post quos* (*t.p.q.*) or *ante quos* (*t.a.q.*). Data structures in `eratosthenes` automatically check for agreement in the ordering of multiple sequences. Other functions include operations to constrain certain sequences to others, which has applications where one wants to determine an optimal ordering of contexts via seriation but has a fixed order for some of the contexts, say, from stratigraphy. In `eratosthenes`, the dates of depositional events are estimated using a two-step version of the Gibbs sampler. From these, the dates of the production, use, and deposition of an artifact-type are estimated. One can also evaluate the chronological "structure" in place, to determine which contexts are pivotal or influential in determining the dates of others.

This paper first starts with an outline of the general approach used in the formal estimation of dates, including the use of contextual seriation as a tool for constructing orderings of unrelated deposits. Seriation, while a mathematical method, is a highly complicated task. No one method will produce the "best" seriation. Hence, optimality measures of the quality of a seriated matrix are essential. Nor will every dataset produce a well-seriated sequence. Here, the *detection* of multiple seriated sequences is advocated, which will elicit the familiar curve or arch in the CA plot (the "Guttman effect": see Fig. 3). Second, definitions of the formal theory and method are given, which are to be sure conventional in probability theory. While Bayesian inference has been a focus, this paper instead places emphasis on the construction of the joint density function, which expresses the relationship between events, to marginalize the date of

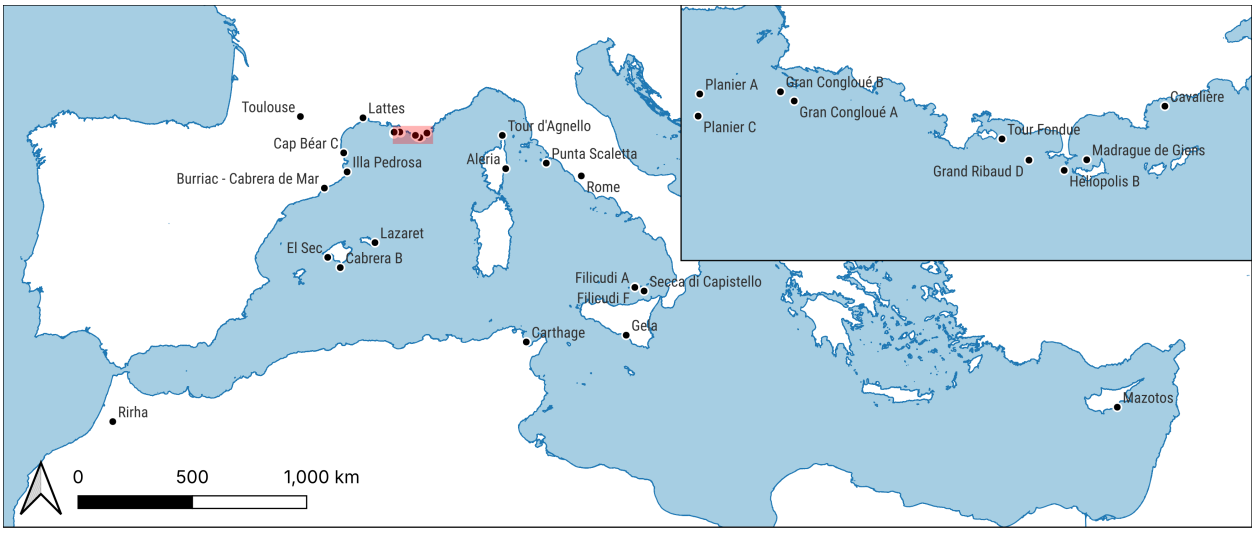


Figure 1: Locations of all sites which provided relative or absolute dating information for this paper.

any one event from that joint density. Seriation affords the means to work sequences established via comparison directly into that joint density. Finally, a case study is undertaken for the date of depositional contexts (including shipwrecks as "deposits"), combining events focusing on the central Mediterranean in the last four centuries BCE. Three core seriations to unify the relationships of various events are developed: funerary assemblages from the necropolis at Aleria; stratigraphic units from the excavation of Quartier 30-35 at Lattes; and selected shipwreck and other terrestrial deposits focusing on black gloss ceramics and amphora types (Fig 1).

2 General Approach

To start, the philosophy behind the development of this particular approach lay in the spirit of generalism and minimalism: the goal is simply to estimate the point in time of some event, given other events that impact or constrain it. Events will either come before or after one another, and so all samples are drawn along the continuum. No period- or phase-labels are used; only the instance of the event and its associated materials, which are used to compare events with one another in sequence. In brief, the process entails:

1. defining a joint conditional probability density function which expresses the potential dates of all contexts on the basis of those sequences, absolute constraints, and their relationships;
2. constructing valid sequences of single and stratified contexts and stipulating *t.p./a.q.*;
3. sampling from that joint conditional density to estimate dates of any one depositional event as a marginal probability density function;
4. deriving production dates of artifact types, per a stated rule about their earliest appearance;
5. sampling any other dates, such as those of use of an artifact, between dates of production and deposition.

Given the constraints of space, this paper focuses on Steps 1-3. The next section thus discusses the definition of the joint conditional density, and explains the marginalization of any one date from that joint density.

2.1 Defining a Joint Conditional Density

The use of probability density functions in archaeological dating has been well established for a number of cases (see e.g. Buck *et al.* 1996; Bronk Ramsey 2015). While probability theory has been applied to select aspects of chronological modeling (e.g., how the p.d.f. of radiocarbon dates changes in light of their stratigraphic sequencing), bringing all sources of dating information into a single joint density function nevertheless requires some comment (the phrase "joint conditional" is sometimes used as essentially synonymous with "joint," since the joint entails the product of multiple conditional probabilities). To start, an event is taken as a variate, X_i, within a joint conditional density $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)$. The marginal p.d.f. of any one event, $f_{X_i}(x_i)$ conveys the probability of the occurrence of X_i in isolation from the other variates, that is to say, regardless of their outcomes. To obtain the p.d.f. of just one event's date, one has to marginalize out the other variates via integration. This marginal p.d.f. of X_i is defined by

$$f_{X_i}(x_i) = \int_{A_n} \cdots \int_{A_1} f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)\, dx_1 \cdots dx_{i-1} dx_{i+1} \cdots dx_n. \tag{1}$$

To give an initial example, one can consider a single sequence of events in order, between a fixed constant *t.p.q.* ϕ and a fixed constant *t.a.q.* ψ, under the condition $\phi < \psi$:

$$\phi \prec X_1 \prec X_2 \prec \cdots \prec X_n \prec \psi$$

where the "$\prec$" symbol indicates "precedes." Given that all X_i represent order statistics of an equal number of uniformly distributed variates on the interval $[\phi, \psi]$, the density of X_i is well known to comprise a four-parameter Beta distribution, with α is equal to i (e.g., $\alpha = 3$ for the p.d.f. of X_3) and $\beta = n - i + 1$ (Johnson *et al.* 1995, 214–215, 280; David and Nagaraja 2003, 9–13):

$$f_{X_i}(x_i; \alpha, \beta, \phi, \psi) = \frac{(x_i - \phi)^{\alpha-1}(\psi - x_i)^{\beta-1}}{(\psi - \phi)^{\alpha+\beta-1}\, \mathrm{B}(\alpha, \beta)} \tag{2}$$

where $\mathrm{B}(\alpha, \beta)$ is the Beta function. It can be highlighted that the Beta distribution represents a generalization of the continuous uniform distribution. That is, on the unit interval $[0, 1]$, $\mathrm{Beta}(1, 1)$ is identical to $\mathrm{Unif}(0, 1)$. It is clear that if there is just one event between ϕ and ψ, such that $\alpha = \beta = 1$, the p.d.f. in Eq. 2 will reduce to the uniform p.d.f, $f_X(x; 1, 1, \phi, \psi) = 1/(\psi - \phi)$. For discussion of the Beta distribution in archaeological applications, which has not received wide attention, see Baxter and Cool (2016).

Beyond a single sequence of order statistics, the marginal p.d.f. becomes more involved, and the definition of the relationships of the events requires more specificity. Take the case where there is just one event X bounded between a uniform variate Φ (as a *t.p.q.*) and another uniform variate Ψ (as a *t.a.q.*), with $\Phi \sim \mathrm{Unif}(r, s)$ and $\Psi \sim \mathrm{Unif}(u, v)$, where r, s, u, v are fixed constants. A straightforward solution for $f_X(x)$ depends not only on ordering of the constants, but also on the independence of Φ and Ψ. Hence, the concept of what "bounded" means above needs to articulated in terms of the dependence of the variates. Given $r < s < u < v$, the case where the constraints Φ, Ψ are independent of X, the distributions

$$X \sim \mathrm{Unif}(\Phi, \Psi), \quad \Phi \sim \mathrm{Unif}(r, s), \quad \Psi \sim \mathrm{Unif}(u, v), \tag{3}$$

mean that event X has no impact on the estimation of the events Φ and Ψ. The case where Φ and Ψ *also* depend on X can be expressed as

$$X \sim \mathrm{Unif}(\Phi, \Psi), \quad \Phi \sim \mathrm{Unif}(r, \min(s, X)), \quad \Psi \sim \mathrm{Unif}(\max(u, X), v), \tag{4}$$

which means that X will affect the estimation of Φ and Ψ, and hence a different marginal density of $f_X(x)$ will result.

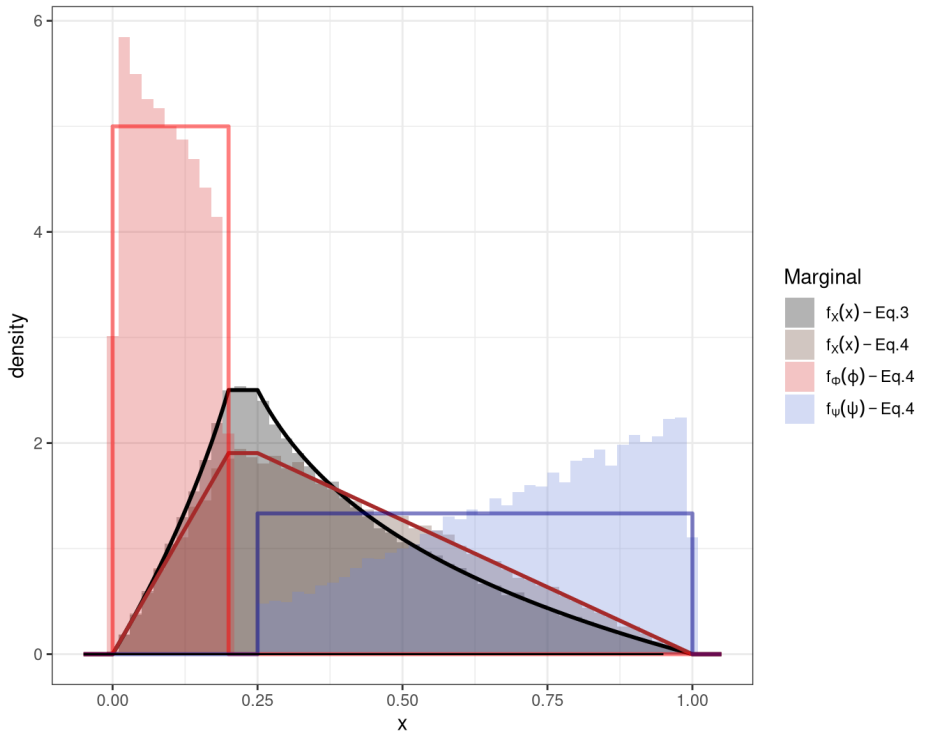


Figure 2: Estimation obtained by simulation of the marginal p.d.f. of $f_X(x)$ with $r = 0, s = 0.2, u = 0.25, v = 1$, given the cases where the absolute constraints Φ, Ψ are either independent of X (Eq. 3) or dependent upon X (Eq. 4) as bounding variates. The black line plots the solution of $f_X(x)$ for Eq. 3, and the brown for Eq. 4, obtained via analysis: see Appendix A.2, A.3, Situation (1), for more details.

The change in density can be illustrated via simulation. Figure 2 shows histograms of samples drawn according to Eq. 3 in comparison with those from Eq. 4. As an analytical solution to marginalize the p.d.f. $f_X(x)$ requires somewhat tedious calculations, details are given in Appendix A. Moreover, direct computation of the marginal p.d.f. poses challenges in certain cases. E.g., if one has a case where $r < u < s < v$, one will need to add the requirement that $\Phi < \Psi$ in order to prevent an impossible situation of an event's *t.p.q.* coming after an event's *t.a.q.* More discussion on this topic can be found in Appendix A, but, in sum, simulation offers an expedient means to approximate densities, as otherwise marginalization will involve proceeding case by case through all possible outcomes and domains of support for howsoever many variates. Moreover, if one is including constraints whose p.d.f. have been determined via simulation (as is typical with calibrated radiocarbon dates), using simulation to estimate any dependent dates will be more practical than, say, fitting a model to the simulated marginal p.d.f.

Therefore, events across multiple sequences can be expressed by a single, joint density by indicating which events precede or succede those in another sequences and by bounding their *termini* at fixed constants or by specified p.d.f. Then, to obtain the p.d.f. of any one date, one marginalizes out all other variates. The package `eratosthenes` developed to these ends implements a Gibbs sampler, a mainstay Markov Chain Monte Carlo technique for deriving a marginal density (Geman and Geman 1984; Smith and Roberts 1993; Buck *et al.* 1996, 188–193). Within a Gibbs sampler, one proceeds by sampling variates one by one, holding all others as fixed. Each variate is updated with a new sample, and then the routine is repeated. The method of Gibbs sampling developed in `eratosthenes` entails a two-stage process, whereby the sampler initializes with the earliest possible value for a *t.p.q.* and latest possible value for a *t.a.q.*, performing an initial, abbreviated

sampling routine upon the addition of each variate in order to avoid a collapse of dates, should a high number of events be inserted into a short interval of time. Then, the main Gibbs sampling routine is run, to achieve an approximate distribution of the marginal p.d.f. (more details may be found below and in the package documentation).

The "statement" of the joint density, as the inputs of the Gibbs sampler, consist of multiple sequences of events. For example, letting α and ω represent the earliest and latest possible events, we can write sequences of events, say, deposits,

$$A \prec B \prec C \prec D \prec E \quad \text{and} \quad B \prec F \prec G \prec H \prec E$$

as a set of uniform variates. In cases where we do not have a known ordering for certain deposits, the relationship is expressed using the operators min and max to specify whichever is the earliest or latest:

$$\begin{aligned} A &\sim \text{Unif}(\alpha, B) & B &\sim \text{Unif}(A, \min(C, F)) & C &\sim \text{Unif}(B, D) \\ D &\sim \text{Unif}(C, E) & E &\sim \text{Unif}(\max(D, G), \omega) & F &\sim \text{Unif}(B, G) \\ G &\sim \text{Unif}(F, E) & H &\sim \text{Unif}(G, E) \end{aligned}$$

On any mth iteration, the Gibbs sampler will proceed by choosing a sample according to each distribution in the following fashion. Here, all samples are chosen uniformly at random. To draw a sample $A^{(m)}$, the routine randomly selects a value between α and $B^{(m-1)}$, which is the sample of B on the previous $(m-1)$th iteration. To draw a sample then of $B^{(m)}$, the routine takes a value from between A^m on the lower bound and then whichever value is the minimum of $C^{(m-1)}$ and $F^{(m-1)}$. Then, for $C^{(m)}$, a random draw is made between $B^{(m)}$ and $D^{(m-1)}$, and so on, until all variates have been sampled, for a total number of M iterations. As noted above, the use of the min and max operators effectively changes the structure of the distribution, "compressing" or "expanding" distributions of the events as order statistics. Consider that each sequence comprises five events, A through E, and B through E. But, stipulating that there are three events between B and E (F, G, and H) means that there are instead six events between the *termini*, increasing the parameter of β (Eq. 2) when it comes to estimating any one event. In the same fashion, the p.d.f. of C and D will be expanded (rescaled) according to the constraints imposed upon B and E.

In defining a joint p.d.f., there is no need to construct a single ordering of all variates. The only necessity is that there are no discrepancies in the ordering of variates across all sequences, since it is impossible for a variate, i.e., an event, to come both before and after itself. Also, care must be taken that the probability densities of the absolute constraints (which can be of any form) should be appropriately designed for their particular sampling situation. For example, if one has multiple intra-sample radiocarbon dates, which measure the decay since death of the same entity, one has effectively a single posterior density as a *t.p.q.*, which is proportional to the product of all p.d.f. of the intra-sampled dates. If there are multiple radiocarbon dates from distinct entities within, say, a soil deposit (and even other datable *t.p.q.*, such as coinage), then the event of the deposition of that context, X, is subsequent to the latest date of those independent *t.p.q.*, i.e., $X \sim \text{Unif}(\max(\Phi_1, \ldots, \Phi_n), \omega)$, where $\Phi_1, \ldots, \Phi_n$ are the variates of all of the *t.p.q.* Such a case can be found below in the incorporation of radiocarbon dates from multiple, distinct organic elements related to the last voyage of the Mazotos wreck (Manning *et al.* 2022; see Table 4 on page 14).

Marginalizing the p.d.f. of a given date from the joint density is thefore the objective, which depends on providing sequences of events. Developing such sequences may be done via judgment on specific cases or positing hypotheses of relationships, or, in the case of archaeological depositional contexts as events, sequences can clearly be obtained from stratgraphic layers (Dye and Buck 2015). The question arises, then, how to relate multiple sequences, which have no physical relationship to one another. In the following section, the argument is made for a return to seriation (ordination) as a method for sequence construction,

relying upon the composition of finds assemblages within contexts, grouping like with like to the greatest extent possible. Again, multiple sequences are possible within the joint conditional: the only necessity is that sequences do not conflict with one another in the ordering of their events. Following a description of the method of seriation used in this paper, examples are given of three "core" seriations which comprise the majority of the scaffolding of the proposed chronology.

2.2 The Principle of Best Comparison: Sequencing and Relating Single and Stratified Contexts

While sequences derived from stratigraphy are generally self-evident, creating sequences from unstratified, single contexts requires a stated process. The same goes for multiple, unrelated, or partial stratigraphic sequences: if one wants to create a single, synthesized ordering of multiple stratified deposits, there needs to be some guiding principle for how they are placed in order. Here, a principle of "proximity by compositional similarity" is invoked: contexts that are most similar in composition to one another (whether in the frequency of types of finds or features, their presence/absence, or any other comparative rationale) are the most proximate to one another in date. In other words, if the same types of finds occur in two deposits, they are closer in time to one another; if they have different types of finds, the more distant in time they are to one another. This principle is synonymous with the technique of contextual seriation, which is the ordering of contexts on the basis of the presence or absence of a type or feature. Contextual seriation has the benefit of being widely applicable across a broader array of evidence, where quantification of find-types is lacking. As an approach, contextual seriation can be viewed as a baseline, applicable to evidence which has not been quantified.

In contextual seriation, a binary matrix is created, with contexts along the rows and types (or more generically, features) along the columns, and in each cell of the matrix, 1 is entered if that type/feature is present in that context and 0 otherwise. Then, the rows and columns are re-ordered to find a most optimal sequence, in which the 1s "move" more or less diagonally as one proceeds down and to the left through the matrix cells (or, up and to the right). There are a variety of techniques available to perform contextual seriation. Here, correspondence analysis (CA) and Procrustes-fit CA were used as they they elicited the most optimal orderings (see Collins-Elliott 2026b). In order to judge the quality of resulting seriations, two criteria of optimality are used: Pearson's correlation coefficient squared (where a higher value indicates a better seriation) or weighted row-column concentration (where a lower value indicates a better seriation).

It should be noted that not every set of contexts and types *can* be well seriated. Contexts may be perturbed or poorly defined, such that they include material from a variety of periods, just as artifact-types may be chronologically spurious (i.e., they may define aspects of an artifact's morphology or function, but those morphological and functional aspects are not sufficiently distinctive from those of other artifacts at other time periods). For example, the Lamboglia 3 black-gloss pyxis has been identified both at the finds assemblage of the shipwreck at El Sec near Majorca (Cerda 1987a, 243), which conventionally dates to the mid 4th c. BCE, as well as in Tomb 60 at Aleria, which conventionally dates to the first half of the 3rd c. BCE (Jehasse and Jehasse 1973, 320 n. 1064). Performing a seriation on materials from the 4th-3rd c. BCE with that type will present a confounded ordering, pulling the ordering of other materials up to a century apart toward one another.

Of course, if stratified contexts are included in the seriation, there is the potential for the seriation to be disordered: the depositional sequences of finds cannot be guaranteed to adhere to the most optimal ordering determined by seriation. To make sure all sequences align in terms of their relative order, the function `sequences()` in the `R` package `eratosthenes` is used to create a `sequences` object, which will work only if no orderings conflict. If it fails, then a diagnostic function `seq_diag()` can be used to identify discrepancies. One can then either remove discrepant contexts and re-run the seriation until `sequences()` passes or use the `seq_adj()` function, which will constrain an ideal, seriated sequence to a fixed,

Table 1: Statistical information on the core seriated matrices, giving references, dimenions $n \times k$ (number of rows and columns), and optimality criteria: the square of Pearson's correlation coefficient (sq.cor.) is the square of the criterion developed by McCormick *et al.* (1969, 147–148); the weighted row-column concentration extends and weights the measure used by Doran (1971). See Collins-Elliott (2026b) for more information. The CA plot shows the initial seriation, unconstrained, while the matrix shows the corrected seriation following the removal of disordered contexts.

			$n \times k$	sq.cor.	w.r.c.conc.
1.	Aleria	Jehasse and Jehasse (1973, 2001)	182×2933	0.87	17.17
2.	Lattes (Corrected)	Py (2004)	107×217	0.68	6.92
3.	Shipwrecks	See Table 3	24×178	0.92	1.87

stratigraphic sequence via piecewise linear interpolation. The `seq_adj()` function is useful for cases of simple misalignments, and this approach can be contrasted with techniques that impose constraints *a priori* to CA (e.g. Poblome and Groenen 2003; van de Velden *et al.* 2009). For more extensive conflicts in a seriation with a stratigraphic sequence, filtering through the seriation to remove discrepant contexts is the more expedient method.

Therefore, one constructs multiple seriations of partial strands (subsets of contexts and find-types), which agree with one another in their ordering. These orderings can be viewed as providing a skeletal or core structure of stratigraphically unrelated contexts, which afford relative constraints for dating all other contexts as much as absolute markers do. Theoretically, it should be possible to construct an entire chronology purely on the basis of relative sequences of events, with only one *t.p.q.* and one *t.a.q.* at the extremes of the sequence, mitigating as much as possible the role played by external events. Such a structure would be tantamount to a chronology based wholly on cultural, social, or environmental features or attributes, without the intervention of political events, such as the destruction of key sites, which are commonly used as absolute markers.

3 Three "Core" Seriations

As a way to evaluate the practical issues that arise in developing cross-project seriations and to offer strategies to deal with them, three different situations were explored. First is the case of burials from the necropolis at Aleria, Corsica, which consist of both single and multi-use tombs. Second, excavations at the site of Lattes in southern France were used to illustrate an approach with contexts which had sequential constraints by virtue of site stratigraphy. Third, in the interest of bringing together multiple single and stratified contexts excavated at different times, the case of shipwreck assemblages along with several other sites were brought together.

The contexts in each of these three situations were seriated using `lakhesis`. In the case of the tombs from Aleria, a more optimal seriation was obtained by Procrustes-fitting the CA scores (on a symmetric biplot) to the polynomial curve. In the other two cases, projecting scores directly onto the first principal axis in simple CA obtain more optimal seriations. Figure 3 illustrates the three seriated matrices on the top row and the appearance of the Guttman effect on the plots of CA/Procrustes-fit CA scores in the second row. Table 1 shows statistical information on each of the seriated matrices, namely their optimality criteria, as less optimal seriations should give way to more optimal orderings (if obtained in the future). Comments about the formation of each of these seriations follow: it should be emphasized that these seriations are very much provisional, given the occasional disagreement with conventional dates, warranting future explanation.

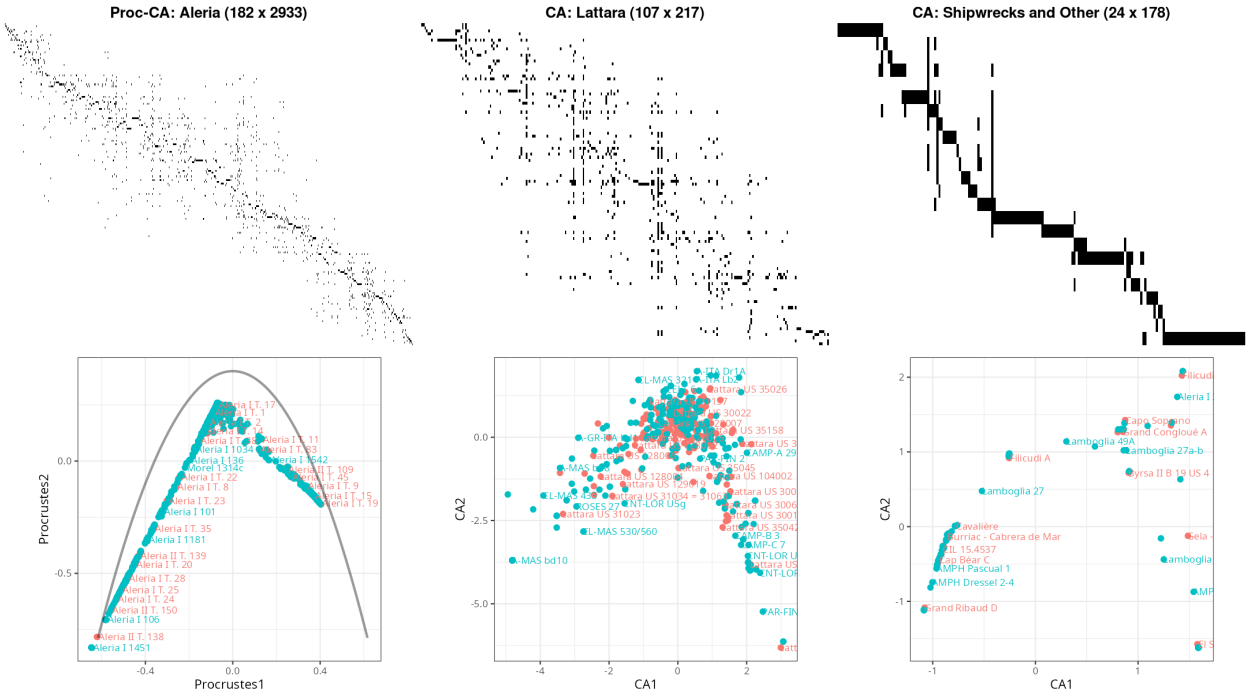


Figure 3: Top row: the seriated matrices of contexts (rows) and find-types (columns) from Aleria, Lattes (ancient Lattara), and a mixture of shipwreck and other assemblages. Bottom row: plots of the CA/Procrustes-fit CA scores used to produce the seriations.

3.1 Contextual Seriation of Tombs at the Aleria Necropolis

The necropolis at Aleria, Corsica, excavated and published by Jehasse and Jehasse (1973, 2001), presents a set of 178 tombs, which span the period from the early 5th century down to the mid-2nd c. BCE. The tombs were initially grouped into three periods (first: 500-350 BCE, second: 350-250 BCE, third: 250-150 BCE). Here, the catalogue number of finds is prefixed with Roman numeral "I" to refer to the first volume (Jehasse and Jehasse 1973), and "II" to refer the second volume (Jehasse and Jehasse 2001). There was only one intrinsically datable *terminus post quem* among the finds, n. I 594 from Tomb 41, a Roman *sextans* (rather than a *quadrans* as reported, since Mercury is on the obverse; *RRC* 190/5), datable to 169-158 BCE. The other coinage attested is Carthaginian. As is customary, the catalogue of the material made reference to comparanda from the necropolis as well as external sources, classifying ceramic forms by conventional typologies (i.e., for black gloss Lamboglia 1952; Morel 1981). In developing a seriation, the objective was to maintain, as much as possible, the rough sequence of the tombs in their original groupings. Since seriation expedites the process of comparison systematically from what is otherwise lengthy, discursive judgment about the relationship of artifacts to each another, it was considered that a successful seriation should not completely upend the general sense of the chronological relationships of finds and types, which is the product of decades of scholarly labor and consideration.

To start, not every tomb from Aleria was included (contexts that comprised redepositional events of earlier finds were avoided). In the course of plotting the CA scores of the data, extreme assemblages were suppressed from consideration (what Cool and Baxter (1999) called "peeling the onion"), to obtain tombs that would elicit a Guttman effect. Not every find-type was included. Direct, one-to-one comparisons with artifacts from other tombs in fact resulted in the clearest seriation, while certain general types tended to produce confused orderings. Among these were Lamboglia forms 11, 21/25, 24, 27, 42A2, 42B, 42A3, 43A,

Table 2: The final twelve tombs according to the seriation of the Aleria necropolis, giving the conventional dates according to Jehasse and Jehasse (1973, 2001) and finds belonging to T. 41 that shifted back its ordering via comparison with those in other tombs.

Aleria I T. 28	ca. 200 BCE	
Aleria I T. 64	ca. 225-200 BCE	
Aleria I T. 60	ca. 200-150 BCE	
Aleria I T. 41	ca. 155 BCE	I 568 compared with I 213 from T. 24 (ca. 200 BCE), Grand Congloué A (ca. later 3rd/early 2nd c. BCE). I 569 compared with I 547 from T. 37 (ca. end of 3rd c. BCE). I 583 compared with I 182 from T. 20 (ca. 225-220 BCE). I 594 gives a *t.p.q.* of 169-158 BCE: see Table 4.
Aleria I T. 62	ca. 250-225 BCE	
Aleria I T. 57	ca. 200-150 BCE	
Aleria II T. 149	ca. 250-225 BCE	
Aleria I T. 25	ca. 200 BCE	
Aleria I T. 24	ca. 200 BCE	
Aleria I T. 61	ca. 200-190 BCE	
Aleria II T. 150	ca. 225 BCE	
Aleria II T. 138	ca. 200-180 BCE	

43A2, and 43B. Progressively suppressing these morphological categories in fact resulted in an ordering where tombs were more proximate to those in their original phase grouping. Comparisons of metal artifacts and other small finds also tended to produce poor seriations, given that these associations were often to tombs in different periods. Keeping the assemblage largely limited to ceramic vessels resulted in the best elicitation of a (highly angular) Guttman effect. In addition, two shipwrecks were added as points of comparanda, El Sec and Grand Congloué A, in order to create an overlap with the third seriation of shipwrecks, as both of these assemblages had some types common to those from the Aleria necropolis.

The most noteworthy shift arose with T. 41. To give some context, Table 2 lists the final twelve tombs of the seriated sequence at Aleria, also giving the finds from T. 41 which resulted in a backward shift in its ordering. Dated to around 155 BCE by Jehasse and Jehasse (1973, 231–235), the intra-site comparisons of its ceramic assemblage in the catalogue largely belonged to tombs from the later 3rd and earlier 2nd c. BCE, resulting in the tomb being pulled earlier in the seriated sequence. This had the equivalent result of pushing many of the tombs that were originally dated to the earlier 2nd c. to a place in the sequence *after* T. 41. Such stated comparisons could not be justifiably dismissed: Jehasse and Jehasse (1973) compared find n. I 568 from T. 41 with I 213 (and also with Grand Congloué A, which conventionally dates to the later 3rd or early 2nd c. BCE), Aleria I 569 with I 547, and n. I 583 with I 182, all of which shifted the tomb earlier in the sequence. All of the tombs that were shifted later, however, were dated generally to the first half of the 2nd c. BCE. The remaining tombs included T. 24 and 25, dated to around 200 BCE, while T. 61 was placed at the start of the 2nd century; it only had three finds. T. 62, 149, and 150 (originally dated 250-225 BCE) all had six or fewer artifacts, making the number of comparisons very low with respect to other assemblages and thereby pushing them to the extreme end of the ordering. Ultimately, though, according the seriation, any tombs after T. 41 should date to the period after 169-158 BCE on the basis of the comparisons in the published catalogue. Alternatively, the date of the coin as a *t.p.q.* needs to be revisited.

3.2 Contextual Seriation of Depositional Contexts at Lattes, Quartier 30-35

Assemblages from excavations at Lattes (ancient Lattara), Quartier 30-35, were included as an example of contexts in stratigraphic sequence (Py 2004). The well-defined systematic typology in DICOCER and DICOCER 2 (Py 1993; Py *et al.* 2001) was used for the ceramic finds. Just as with the assemblages from Aleria, contexts whose CA scores were at an extreme distance on the biplot were suppressed, as were contexts that did not contribute to the emergence of a Guttman effect. From those contexts that remained, common find-types that were suffuse throughout the stratigraphic sequence were omitted to discern the most chronologically determinative types. Omitted types included the following: A-AFR TrA; A-ITA Dr1; A-PUN C2a; CAMP-A 6, 25, 27a-b, 27c, 27Bb, 28ab, 31b, 36; CL-REC 1, 2, 3, 13; CNT-LOR C2, C3, C4, J1a, J1b, J1c, J4a, V2a, V2b, U7; COM-IT 8e; and DOLIUM bd8f. Such find-types were not removed entirely from the analysis, but merely temporarily suppressed to obtain an optimal seriation.

The resulting contextual seriation, while presenting an "optimal" ordering, did not agree well with the stratigraphic ordering of contexts. Often, such misalignments were very proximate (i.e, a context was placed just before when it should have come just after), but occasionally there were deposits that occured far out of phasing. These issues were to be expected: there will be deposits that are "outliers" in the sense that they contain a few find-types and possibly even entirely residual material. Surface contexts especially will be prone to contain a chronologically broad array of materials. While the `seq_adj()` function from `eratosthenes` can adjust simple cases of such misalignments, the way in which multiple contexts were out of order made it more expedient to filter through the contexts, identifying and removing contexts from the seriation that violated the stratigraphic sequence. The resulting seriation contained 107 contexts from the set of 667 archaeological events that were entered in from the stratigraphic sequences and activities, as illustrated in Py (2004, Figs. 17, 37, 74, 97, 204, 230, 243, 267, 295, 317, 346, 382, and *passim*).

3.3 Contextual Seriation of Shipwreck Assemblages

Finally, a third seriation was produced based on shipwreck assemblages from the following wrecks, which span the 4th through 1st centuries BCE: El Sec, Filicudi (Capo Graziano) A and F, Secca di Capistello, Lazaret, Cavalière, Punta Scaletta, Grand Congloué A and B, Madrague de Giens, Planier C, Cap Béar C, and Grand Ribaud D (Table 3). Additional contexts comprised of amphora types were added, such as the silo at Burriac (Cabrera de Mar, Barcelona, UE 2022), which included a consular date on a Dressel 1B amphora in its assemblage, as well as several depositions of amphorae from Toulouse (Veille-Toulouse Puits 4 and 5, Fosse 40, and Grand Citerne; Toulouse Caserne Niel and Coteaux d'Estarac). To illustrate the use of artifacts or contexts that provided important absolute dating information with only one or two finds, two contexts were added: first is the incidence of a Greco-Italic amphora stamp, EYΞENY, from the house excavated at Via Polieno, Gela (Blanck 1978, 96–97; Orlandini 1956); the second is a Dressel 1B from Castro Pretorio, Rome, which had a consular date identical to *CIL* 15.4537 (see the explanation in Tchernia 1986, 320 n. 10). Also included were amphorae from Capo Soprano, Gela, which Benoit (1961, 40) had compared to those from Grand Congloué A, even though their context remains to be checked for whether it pre- or post-dates the destruction of Gela in 282 BCE.

The sequences of the shipwrecks were already related to those from Aleria by virtue of the El Sec and the Grand Congloué A wrecks. In order to relate these assemblages to those from Lattes, the assemblages belonging to Grand Congloué A, Veille-Toulouse - Puits 5, and Carthage Byrsa II B 19 US 4 were inserted, one at a time, into the dataset from Lattes, Quartier 30-35, to generate three separate seriations. As these three assemblages had much overlap in terms of their find-types, including them all at once into the data from Lattes would collapse the curve of the Guttman effect and confuse the optimal seriation already obtained for that site. Hence, adding them in individually prevented this confusion from happening, such that shorter,

Table 3: Contexts selected for the third seriation, comprising mostly shipwreck assemblages with additional terrestrial contexts which contained transport amphorae and black gloss. See also Vandermersch (1994), Olcese (2004, 2005), Cibecchini and Capelli (2013), and Loughton (2014) for discussion on the Greco-Italic and Dressel 1 family of types, subtypes, and transitional types.

Burriac - Cabrera de Mar (UE 2022)	after 90 BCE	Miró (1991).
Byrsa II B 19 US 4	mid 2nd c. BCE	Lancel *et al.* (1982, 196–198).
Cap Béar C	ca. 50-25 BCE	Liou and Pomey (1985, 547–551); Parker (1992, 97–98).
Capo Soprano, Gela	—	Benoit (1961, 40) connects amphorae from Grand Congloué A to this context, but the precise assemblage remains to be checked as it could post-date the destruction of Gela (Vandermersch 1994, 73, 209 n. 143).
Cavalière	ca. 100 BCE	Charlin *et al.* (1978, 48 n. 6); Parker (1992, 133–134).
CIL 15.4537	after 97 BCE	Tchernia (1986, 320 n. 10).
El Sec	ca. 360-340 BCE	Cerda (1987b, 1987a); Parker (1992, 392–394).
Filicudi (Capo Graziano) A	ca. 160-140 BCE	Cavalier (1985); Parker (1992, 117).
Filicudi (Capo Graziano) F	ca. 300-250 BCE	Bernabò-Brea *et al.* (1985); Parker (1992, 118–119).
Gela - Casa Via Polieno	before 282 BCE	Blanck (1978, 96–97); Orlandini (1956).
Grand Congloué A	ca. 210-180 BCE	Benoit (1961) and Long (1987); Parker (1992, 200–201).
Grand Congloué B	ca. 110-80 BCE	Long (1987); Parker (1992, 201).
Grand Ribaud D	ca. 10-1 BCE	Parker (1992, 203–204); Hesnard *et al.* (1988).
Lazaret	late 3rd-early 2nd. c. BCE	Fernández-Miranda *et al.* (1977, 83–94); Parker (1992, 241).
Madrague de Giens	70-50 BCE	Liou and Pomey (1985, 559–567); Parker (1992, 249–250); Tchernia (1978).
Planier C	ca. 60-40 BCE	Liou and Pomey (1985, 553–556); Parker (1992, 316–317).
Punta Scaletta	ca. 140-130 BCE	Lamboglia (1964); Parker (1992, 359).
Secca di Capistello	ca. 300-280 BCE	Blanck (1978, 96–97); Frey *et al.* (1978); Parker (1992, 396).
Toulouse, Caserne Niel - Puits 5	mid-late 1st c. BCE	Benquet (2013, 140–142).
Toulouse, Coteaux d'Estarac - Puits 8	1st third of 1st c. BCE	Benquet (2013, 142–144).
Veille-Toulouse - Fosse 40	1st c. BCE	Benquet (2013, 146).
Veille-Toulouse - Grande Citerne	2nd half of 1st c. BCE	Benquet (2013, 154–156).
Veille-Toulouse - Puits 4	1st c. BCE	Benquet (2013, 144–145).
Veille-Toulouse - Puits 5	2nd half of 1st c. BCE	Benquet (2013, 152–154).

partial strands of seriated sequences which included contexts both from Lattes and those sites were obtained, which agreed with the optimal seriation and the stratigraphic sequences from those sites.

Another short stratigraphic sequence of Byrsa II B 19 US 4 $\prec$ Byrsa II B 19 2 was added (B 19 2 was not included in the seriation). Additional relationships were added as "hypotheses," either relative sequences that agreed with the conventional dating of shipwrecks or absolute constraints which were necessary to provide definitive *t.p./a.q.* for the sequences. These hypotheses are:

- The deposition of Byrsa II B 19 2, datable to before 146 BCE, occurred between the shipwrecks at Lazaret (conventionally ca. 160-140 BCE) and Punta Scaletta (conv. ca. 140-130 BCE).
- The wreck at El Sec occurred after the wreck at Mazotos, Cyprus, the latter of which has several radiocarbon dates associated with its final voyage as *t.p.q.* located in the earlier 4th c. BCE.
- The tombs in the seriation from Aleria are bounded between the dates of 500 and 146 BCE.
- The deposition of Lattara US 128004 dates to after 225 BCE and the deposition of US 35236 occurred in the interval of 25-1 BCE.

- The deposition of amphorae in UE 2022 at Burriac, Cabrera de Mar, occurred before 85 BCE (five years after the consular date on the Dressel 1B amphora).
- Rirha US 5182 $\prec$ Rirha US 5154 (Callegarin *et al.* 2016, 40–41). There is no direct stratigraphic relationship, but Rirha US 5182 is considered to date well before US 5154. US 5154 in turn is hypothesized to have been deposited before the Planier A shipwreck.
- The following sequences of shipwrecks hold:
  - El Sec $\prec$ Filicudi F $\prec$ Tour Fondue $\prec$ Cabrera B $\prec$ Tour d'Agnello $\prec$ Sanguinaires A $\prec$ Lazaret.
  - Cabrera B $\prec$ Grand Congloué A $\prec$ Lazaret $\prec$ Punta Scaletta $\prec$ Illa Pedrosa $\prec$ Cavalière $\prec$ Madrague de Giens $\prec$ Planier C $\prec$ Planier A.
  - Grand Congloué A $\prec$ Héliopolis B $\prec$ Punta Scaletta.
- The conventional dates of the wrecks Grand Ribaud D (ca. 10-1 BCE) and Planier A (ca. 1-15 CE) were used as necessary *t.a.q.* for their deposition.

A total of 217 sequences (both stratigraphic and seriated) were therefore defined, contained in a `list` object for input into the `gibbs_ad()` function in `eratosthenes`, which estimated the date of deposition (for the code, see the appendix to this paper).

4 Absolute Constraints

Having synthesized all sequences and checked them for agreement, absolute constraints were then entered as either *t.p.q.* or *t.a.q.* The *t.p.q.* are shown in Table 4, consisting of calibrated radiocarbon dates, coinage, consular dates (painted on the exterior of amphorae), or the aforementioned hypotheses. Radiocarbon dates were calibrated via `BChron`, and so samples from their density functions were incorporated directly into the `list` object of *t.p.q.* available in the supplementary files to this paper.

The *t.a.q.*, shown in Table 5, consisted of two destruction events, the sacks of Gela in 282 BCE and Carthage in 146 BCE. Additional *t.a.q.* included hypotheses in order to fix the endpoints of analysis with the wrecks of Grand Ribaud D and Planier A, which respectively occurred toward the end and at the start of the BCE/CE millennial transition.

5 Results

In total, 429 depositional events and 33 external constraints (*t.p./a.q.*) were furnished for this exercise. Running the `gibbs_ad()` function from the package `eratosthenes` yielded an estimate of the marginal p.d.f. for each event, all of which are given in Table S1, as a supplementary file. In this function, convergence in distribution is assessed using consistent batch means (CBM), which divides the total number of simulation samples taken up to a given point, denoted n^*, into batches of equal size, here $\sqrt{n^*}$. For each event, the mean of those batches is computed, along with the standard error pursuant to the number of batches, that is, the Monte Carlo standard error (MCSE). If the mean MCSE falls below a specified criterion (by default 0.5, to determine the date of an event +/- 1 year), the main Gibbs sampler will stop. Jones *et al.* (2006) and Flegal *et al.* (2008) discuss the statistical details of this process. Given that this is the mean MCSE of all events, certain events will have higher or lower MCSE. Likewise, even though the mean of the marginal distribution is used to assess convergence, it will not necessarily be the most probable outcome, which can be determined using a highest density region (HDR; Hyndman 1996).

Some comments can be made about the results in general. Figure 4 shows marginal density histograms of twelve selected dates of depositional events, and Table 6 shows their mean, MCSE, and bounds of the 10% HDR (the summary results of all events can be reproduced by executing the R code in the supplementary files). The marginal density of the date of the El Sec wreck has been pushed earlier than its conventional date,

Table 4: *T.p.q.* applied to the contexts in the paper. P.d.f. of the calibrated radiocarbon dates were estimated using the `Bchron` package (Haslett and Parnell 2008) with the IntCal20 atmospheric data (Reimer *et al.* 2020).

Aleria I T. 19	Hypothesis	500 BCE	
Mazotos	VERA-6082a	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	VERA-6082b	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	VERA-6082c	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	VERA-6082d	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-31836	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-31877	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-32005	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-32006	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-32794	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-32795	cal. r.c. p.d.f.	Manning *et al.* (2022).
Mazotos	OxA-32796	cal. r.c. p.d.f.	Manning *et al.* (2022).
Rirha US 5182	RH A08 US 5182	cal. r.c. p.d.f.	Callegarin *et al.* (2016, 41).
Lattara US 128004	Hypothesis	225-200 BCE	Py (2004, 71–73).
Filicudi A	Coin - *RRC* 142/1	189-180 BCE	Cavalier (1985, 107–108).
Aleria I T. 41	Coin - *RRC* 190/5	169-158 BCE	Jehasse and Jehasse (1973, 235 n. 594)
Punta Scaletta	Coin - Ptolemy VI Philometor	180-145 BCE	Lamboglia (1964).
Veille-Toulouse - Fosse 40 sup.	Cos. date on Dr. 1A	103 BCE	Benquet (2013, 146).
Dressel 1B - Castro Pretorio	Cos. date identical to *CIL* 15.4537	97 BCE	Tchernia (1986, 320 n. 10).
Burriac, Cabrera de Mar	Cos. date on Dressel 1B	90 BCE	Miró (1991, 57).
Madrague de Giens	Coin - *RRC* 236	137 BCE	Tchernia (1978, 15–16).
Madrague de Giens	Coin - *RRC* 382	79 BCE	Tchernia (1978, 15–16).
Madrague de Giens	Coin - *RRC* 392	75 BCE	Tchernia (1978, 15–16).
Cavalière	Coin - Mazard 50	202-88 BCE	Charlin *et al.* (1978, 48 n. 6).
Cavalière	Coin - Mazard 45	202-88 BCE	Charlin *et al.* (1978, 48-49 n. 7).
Cavalière	Coin - Mazard 41	206 BCE	Charlin *et al.* (1978, 49 n. 10).
Cavalière	Coin - Carteia M SEP / KAR	101 BCE	Charlin *et al.* (1978, 50-51 n. 11).

Abbreviations: *t.p.q.* = *terminus post quem*; cal. r.c. = calibrated radiocarbon; p.d.f. = probability density function; cos. = consular; *RRC* = Crawford (1974); Mazard = Mazard (1955).

Table 5: *T.a.q.* applied to the contexts.

Gela - Casa Via Polieno	Destruction of Gela	282 BCE	Blanck (1978, 96–97); Vandermersch (1994, 166).
Byrsa II B 19.2	Destruction of Carthage	146 BCE	Lancel *et al.* (1982, 194).
Aleria II T. 138	Hypothesis	146 BCE	
Burriac, Cabrera de Mar	Hypothesis	85 BCE	
Lattara US 35236	Hypothesis	25-1 BCE	Py (2004, 182).
Grand Ribaud D	Hypothesis (necessary endpoint)	10-1 BCE	Hesnard *et al.* (1988, 145).
Planier A	Hypothesis (necessary endpoint)	1-15 CE	Parker (1992, 315); Corsi-Sciallano and Liou (1985, 17–25).

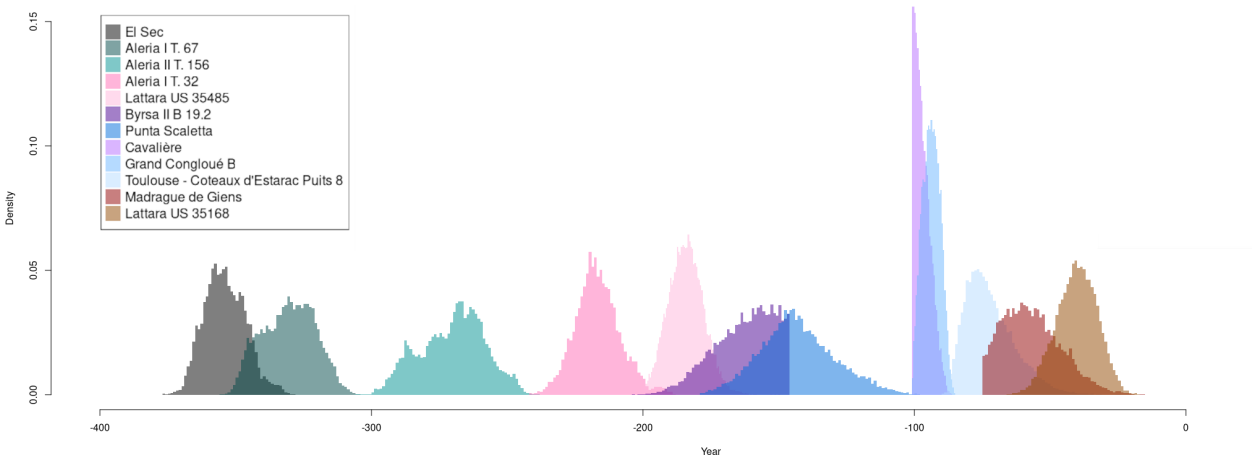


Figure 4: Density histograms of the marginal p.d.f. of twelve select depositional events. Results for all 429 sequence events and 33 external constraints are given in (see Table S1, provided in a supplementary pdf, `caa2025_table_s1.pdf`).

which can be attributed in part to the number of tombs in the sequence at Aleria. Of the tombs displayed in the histogram, the irregularly shaped density is owed to the hypothesis that the El Sec wreck comes after the Mazotos wreck, whose *t.p.q.* is determined by radiocarbon dates, such that the probability densities of those radiocarbon dates carry over into all subsequent events. Aleria T. 67 was conventionally dated to ca. 350-320 BCE, T. 156 to around 300 BCE, and T. 32 to the second quarter of the 3rd c. BCE. The date of T. 67 largely agrees with its conventional dating, but both T. 32 and T. 156 have been pushed later. Lattara US 35485, conventionally 150-125 BCE, likewise has been greatly shifted, this time earlier. The deposition of Byrsa II B 19 US 2, whose material is largely 2nd c., reaches its maximum probable date of deposition just before the *t.a.q.* of the destruction of Carthage in 146 BCE. The wreck of Punta Scaletta, which was hypothesized to come after Byrsa II B 19 US 2, lies within its conventional date range, as do the wrecks of Cavalière, Grand Congloué B, and Madrague de Giens. Lattara US 35168, which was dated to 100-75 BCE, has been pushed later by around half a century. The deposition of Puits 8 at Coteaux d'Estarac also agrees with the conventional date. It should be stressed that these results represent an initial attempt, which can and will be revised.

While it was desirable to maintain conventional dating, shifts in dates are to be expected. At Aleria, the case of T. 41 was already mentioned. In general, the seriation of the tombs tended to be placed at equal intervals from 500 to 146 BCE in the absence of additional constraints. The case of the seriated deposits from Lattes showed larger departures from their conventional phase dates given by Py (2004), and so more work will be required to comprehend why such discrepancies have occured for dates of deposits at Lattes. One consideration is that contextual seriation may not be ideal for some stratigraphic sequences. After all, in each context, there might be an amount of residual material which, even if a small quantity, is registered as a "1," just as much as a large quantity of material would be. With more residual material, such a seriation would present more confusion than closed, primary deposits like tombs and shipwrecks (although here too there are cases of mixed and comingled assemblages, it should be stressed). Instead, harmonizing stratigraphic sequences may benefit more from the identification of certain widespread or diffuse find-types, whose earliest appearance at a site constitutes a *t.p.q.* and whose earliest absence serves as a *t.a.q.* Such a method is, more or less, in line with what archaeologists already do in forming chronologies, but here investigators must formally state such hypotheses, which, as a rule, can be applied universally across the entire joint density (i.e., not relying on one find-type for one set of contexts, and another find-type for another).

It can be noted also that the single event of the deposition of the Dressel 1B amphora from Castro Pretorio,

Table 6: Mean, MCSE, and lower and upper bounds of the 10% highest density region (HDR) of the marginal densities illustrated in Figure 4. Negative dates indicate BCE.

	mean	MCSE	10% HDR (lower)	10% HDR (upper)
El Sec	-354.0	0.60	-358.5	-353.1
Aleria I T. 67	-330.6	0.80	-331.9	-320.6
Aleria II T. 156	-270.3	1.01	-269.0	-261.5
Aleria I T. 32	-216.7	0.67	-221.1	-215.6
Lattara US 35485	-184.2	0.46	-187.3	-182.3
Byrsa II B 19.2	-162.7	0.23	-159.2	-147.0
Punta Scaletta	-143.0	0.19	-149.2	-141.5
Cavalière	-97.1	0.04	-100.9	-99.4
Grand Congloué B	-93.8	0.05	-95.4	-91.8
Toulouse - Coteaux d'Estarac Puits 8	-72.1	0.23	-80.3	-73.8
Madrague de Giens	-56.7	0.33	-65.1	-53.8
Lattara US 35168	-40.2	0.44	-42.4	-37.5

Rome, had a mean MC date of 43/42 BCE (MCSE 0.36), but the consular date on the amphora was from 97 BCE. The reason for this downdating is that that find consists of a single Dressel 1B amphora, and therefore the contextual seriation placed it in a sequential position that is effectively at the "central tendency" of that artifact type, given all other contexts that contain the Dressel 1B. If this dating is undesirable, the investigator is able to impose their own *t.a.q.* on the event of the deposition of that amphora.

Ultimately, the benefit of a unified, formal framework for chronology as proposed here is that one obtains precise, numerical estimates for events, taking into account *all* events for their impact on one another. Any date can be expressed, just as with radiocarbon dates, as a highest density region (HDR), which gives the range of the most probable occurrences of an event. Taking the Madrague de Giens shipwreck, whose mean estimated date is 57/56 BCE (MCSE 0.33), a narrow range can be taken with a 10% HPD, which covers 65-53 BCE. A broad estimate can be taken with a 95% HPD, which spans the period from 75-15 BCE. Such a date is affected not just by the immediate *t.p.q.* of the coinage on board, but its typogological comparisons with all other events in its seriation. A formal approach also has the benefit in allowing one to inspect discrepancies with the conventional chronology, to determine if the causes of disagreements lay in the typological structure of the categories used, the lack of sufficient constraints, or even in the need to insert more hypothetical assumptions. Or, there may be no justifiable excuse for the conventional dating, in which case it should be abandoned.

6 Conclusion

This paper has introduced the means and methods to establish chronologies entirely in probabilistic terms directly from the act of comparison, via seriation. The quality of a seriation is conveyed via optimality criteria, such that the most optimal ordering can be chosen, and sequences of contexts can be harmonized with other relative sequences in agreement with fixed, stratigraphic relationships. The continuous uniform distribution offers a default expression of uncertainty for the occurrence of one event between any two others, and absolute constraints are added as variates themselves, resulting in a joint conditional density from which any one date can be marginalized. The main hurdle lies in the question of how to associate unrelated contexts with one another. Here, contextual seriation was advocated as a baseline to measure the similarity of contexts to one another in terms of their finds assemblages. As discussed, frequency seriation may be more germane for the case of seriated stratigraphic contexts, or any contexts which may include a high number of low

counts of residual or sporadic material. However, instead of constructing a single "grand seriation," this paper advocates for developing multiple, partial, "most optimal" seriations, which are constrained to stratigraphic relationships. There is no need for phasing or periodization in terms of determining the date of an assemblage. Events can be directly placed into sequence via seriation, on the basis of the principle of comparison.

The formal chronology developed here that unifies the relationships among depositional events of the central Mediterranean ca. 4th-1st c. BCE is provisional, needing a fuller review and the addition of more contexts and typological associations. The relationships established between artifacts that are formalized via seriation are not novel ones, but the impact of their logical consequences on all other dates can *only* be evaluated against one another in their entirety via a formal approach. Results from this method are sure to differ from conventional chronology, since the full balance of how inter-assemblage comparisons and absolute markers affect one another quantitatively has heretofore not been assessed.

Further information and examples are available in the documentation of the R packages `lakhesis` and `eratosthenes`, which offer more functionality than what can be presented in this paper (including dating the production, use, and deposition of artifact types). Users are kindly requested to direct any comments to the author for package improvements.

Supplementary Materials

Data and code are available online (https://doi.org/10.5281/zenodo.17508910), in either `.rda` or `.csv` and plain-text `.r` formats, with supplementary Table S1 uploaded as a pdf file:

- `caa2025_table_s1.pdf` contains the mean and Monte Carlo standard error of all 462 events (429 sequential events and 33 absolute constraints).
- `caa2025_ser20251001.rda` contains the three seriations produced in this paper, which are also downloadable in the following `.csv` formats as seriated matrices (see Table 1):
  - `caa2025_ser_aleria.csv` contains the seriated matrix obtained for the tombs at Aleria.
  - `caa2025_ser_lattara.csv` contains the seriated matrix obtained for stratigraphic contexts at Lattes.
  - `caa2025_ser_shipwrecks.csv` contains the seriated matrix of the core seriation of shipwrecks and other associated contexts.
- `caa2025_eda20251001.rda` contains all data regarding stratigraphic sequences and absolute constraints. This information was keyed in manually, and the following plain-text `.r` file contains the text to produce these sequences and constraints.
- `caa2025_script.r` is a plain text script containing all R code used to produce the results of this paper, including the inputs of the stratigraphic sequences and absolute constraints.

As `.rda` files are `R`'s interchange format, objects are loaded into memory already named. Sequences are contained in the `list` object named `contexts_20251001`, *t.p.q.* are contained in `tpq_20251001`, and *t.a.q.* in `taq_20251001`.

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Data Availability Statement

The data that support the findings of this study are openly available in Zenodo at https://doi.org/10.5281/zenodo.17508910.

Conflicts of Interest

The author declares no conflicts of interest.

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A Appendix A: Some Analytical Solutions to Marginal p.d.f. of Dates

The following appendix presents some analytical solutions to the marginal p.d.f. of dates in sequences. As noted above, Monte Carlo techniques are commonly used in the estimation of p.d.f. where integration is difficult or intractable. Direct computation via analysis is preferable, though. Where one only has order variates (events in sequences) which are bound by constants, an analytical solution to the marginal p.d.f. will be tractable, but, depending on the structure of the joint conditional, it will be of varying levels of difficulty. Strategies include the use of Fubini's theorem, as well as polynomial division, integration by partial fractions, and integration by parts, in order to turn the product of the joint density into a sum of separate terms. Attention to the domain over which one is integrating is also crucial, with respect to the sequence of the constraints furnished, in order to avoid impossible outcomes (e.g., a variate which serves as a *t.p.q.* for others coming before another that is its *t.a.q.*). Incorporating analytical solutions into the functionality of `eratosthenes` is a current goal.

A.1 Multiple Events within Fixed Termini

The simplest case, of a single sequence of events between two fixed constants ϕ (as a *t.p.q.*) and ψ (*t.a.q.*) has its marginal p.d.f. given in Eq. 2, where α is the number of the event i and β is equal to $n + 1 - i$, where n is the total number of events.

A.2 Single Event within Variable (Uniform) Termini, Independent of the Event

The situation of a single event bound between two uniformly random variates presents a more complicated situation, but an analytical solution exists. Let the *t.p.q.* Φ and the *t.a.q.* Ψ be uniformly distributed each between two constants, and there is only one variate, X, which is uniformly distributed between Φ and Ψ. The p.d.f. of X is therefore $f_{X|\Phi,\Psi}(x|\phi,\psi) = 1/(\psi - \phi)$. Let the *t.p.q.* lie between fixed constants r and s, with $r < s$, and the *t.a.q.* lie between u and v, with $u < v$. That is, $\Phi \sim \text{Unif}(r, s)$ and $\Psi \sim \text{Unif}(u, v)$. Given independence in the variates of Φ and Ψ, one has $f_{\Phi,\Psi}(\phi,\psi) = f_{\Phi}(\phi) f_{\Psi}(\psi)$. That

is, the selection of a value for Φ does not affect the selection of a value for Ψ, and vice versa. The solution to the double integral that marginalizes out ϕ and ψ to obtain $f_X(x)$ is:

$$f_X(x) = \iint_A f_{X|\Phi,\Psi}(x|\phi,\psi) f_\Phi(\phi) f_\Psi(\psi)\, d\phi\, d\psi. \tag{5}$$

In solving for the definite integral, the region of A requires careful attention, since it will change based upon the ordering of the fixed constants with respect to one another and to the marginal variate (here just X). While it had initially been defined that $r < s$ and $u < v$, such a definition results in six different situations to consider for these constants. The first and last of these situations are straightforward to solve, but the others present difficulties insofar as they result in a situation where Φ and Ψ are dependent upon one another.

(1) $r < s < u < v$. To simplify the following discussion, the notation $A = [r, s] \times [u, v]$ is abused to represent the domain $A = \{(\phi, \psi) \in \mathbb{R}^2 : r \le \phi \le s,\ u \le \psi \le v\}$. For this domain of A, the solution of the double integral in Eq. 5 can be made more concise by defining the following function:

$$g(r, s, u, v) := \frac{1}{(s-r)(v-u)} \log\left[\frac{(v-r)^{v-r}(u-s)^{u-s}}{(v-s)^{v-s}(u-r)^{u-r}}\right].$$

The marginal p.d.f. $f_X(x; r, s, u, v)$ will involve integrating over three different domains of A. Starting from Eq. 5, the marginal will take the form

$$f_X(x) = \iint_A \frac{1}{\psi - \phi} \frac{1}{s-r} \frac{1}{v-u}\, d\phi\, d\psi.$$

Applying basic rules of integration for the reciprocal of terms and logarithms, one obtains

$$f_X(x) = \begin{cases} g(r, x, u, v) & \text{for } r \le x < s; A = [r, x] \times [u, v] \\ g(r, s, u, v) & \text{for } s \le x < u; A = [r, s] \times [u, v] \\ g(r, s, x, v) & \text{for } u \le x < v; A = [r, s] \times [x, v] \end{cases}$$

This result is illustrated by the black line in Figure 2.

(2) $r < u < s < v$, which cannot tolerate a sample where $\psi \le \phi$ and hence results in dependence between Φ and Ψ (see Section A.3);

(3) $r < u < v \le s$, which must imply instead a case where $s = v$, and again involves dependence between Φ and Ψ to ensure that $\psi < \phi$ (see Section A.3).

(4) $u \le r < s < v$, which similarly must imply that $r = u$ and again involves dependence between Φ and Ψ (see Section A.3).

(5) $u \le r < v \le s$, which must imply instead a case where $r = u$, $s = v$ for the same reason. This is identical to the situation of Example A.1, in which one has three ordered events (Φ, X, Ψ) bound between two fixed *termini*.

(6) $r < s = u < v$. This case will yield the same form of the p.d.f. as Situation (1), only now as $s = u$, we have:

$$f_X(x) = \begin{cases} g(r, x, u, v) \text{ for } r \le x < s; A = [r, x] \times [u, v], \\ g(r, u, x, v) \text{ for } u \le x < v; A = [r, u] \times [x, v]. \end{cases}$$

Discussion of these situations where there is dependence among the constraints are discussed in the following section.

A.3 Single Event within Variable (Uniform) Termini, Dependent upon the Event

Each of the situations in the previous section have counterparts where constraints are dependent upon X, i.e., with $\Phi \sim \text{Unif}(r, \min(s, X))$ and $\Psi \sim \text{Unif}(\max(u, X), v)$, as given in Eq. 4. In Situations (1), (5), and (6), the marginal p.d.f. is straightforward. But for Situations (2-4), I regret that time has prohibited me from further pursuit of a solution. As can be seen in the Section A.4, even when analytical solutions are possible thanks to the presence of independent variates, the computations can be somewhat labor-intensive.

(1) This case will result in a trapezoidal density, which has been studied by van Dorp and Kotz (2003) and Lee and Bronk Ramsey (2012). The marginal p.d.f. of X will be:

$$f_X(x) = \begin{cases} \dfrac{2(x-r)}{(s-r)(v+u-s-r)} & \text{for } r \le x < s, \\ \dfrac{2}{(v+u-s-r)} & \text{for } s \le x < u, \\ \dfrac{2(v-x)}{(v-u)(v+u-s-r)} & \text{for } u \le x \le v. \end{cases}$$

This result is illustrated by the brown line in Figure 2. Likewise, one can marginalize X, Ψ to obtain the marginal p.d.f of Φ, and Φ, X to obtain that of Ψ.

(2-4) These situations involve the order statistics of dependent variates. As the Beta density in Eq. 2 has been derived under the assumption that the occurrences of events are independent and identically distributed (i.i.d.), the non-i.i.d. case presents more complications when events are dependent and/or not identically distributed. David and Nagaraja (2003, 95–112) covers these situations (see also Maurer and Margolin 1976). One would need to work through the combinatorics of events with regard to their cumulative distribution function (c.d.f.), and then take their derivative of the c.d.f. to yield a p.d.f.

(5) This case again presents a situation equivalent to the p.d.f. of Eq. 2.

(6) The case of Situation (6) above results in a triangular distribution when the constraints are dependent upon X, as a reductive case of the trapezoidal distribution in Situation (1). Where $s = u$, we have

$$f_X(x) = \begin{cases} \dfrac{2(x-r)}{v-u}\left(\dfrac{1}{r-v} - \dfrac{1}{r-u}\right) & \text{for } r \le x < u, \\ \dfrac{2(x-r)}{v-u}\left(\dfrac{1}{r-v} - \dfrac{1}{r-x}\right) & \text{for } u \le x \le v. \end{cases}$$

A.4 Multiple Events within Variable (Beta) Termini, Independent of the Events

Marginalizing the order statistics for multiple events in a single sequence, that is, deriving the marginal p.d.f. of $f_{X_i}(x_i)$ bound between two uniform variates Φ and Ψ, gives way to the more general problem where Φ and Ψ are themselves four-parameter Beta distributions, given that the continuous uniform is just a special case of the Beta. Such a generalization means that one can give the formal equations for any one event, which may be the result of multiple *t.p.q.* and *t.a.q.* across multiple sequences. Hence, Eq. 1 will integrate the product of multiple Beta distributions that comprise the joint density. The greater the number of sequences and external constraints that are included, the more laborious the integral becomes for any one variate.

The following example illustrates a simple case of an order variate in one sequence which depends on another in a second sequence. The first sequence contains three events bound by two fixed constants, $\phi_1 < X_1 < X_2 < X_3 < \psi_1$. The second sequence is bound by X_2 and the fixed constant ψ_2, such that $X_2 < Y_1 < Y_2 < Y_3 < \psi_2$. It is required that $\phi_1 < \psi_1 < \psi_2$ to ensure independence. The marginal

p.d.f. of, say, Y_2 can be obtained analytically. Knowing that X_2 and Y_2 follow a Beta distribution, since they are order variates, $X_2 \sim \text{Beta}(\alpha_1, \beta_1)$ and $Y_2 \sim \text{Beta}(\alpha_2, \beta_2)$, with $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 2$ (given that they are both the second out of three total events). The marginal will therefore be the product of two four-parameter Beta distributions:

$$\begin{aligned} f_{Y_2}(y_2) &= \int_A f_{Y_2|X_2}(y_2|x_2) f_{X_2}(x_2)\, dx_2 \\ &= \int_A \frac{(y_2 - x_2)^{\alpha_2 - 1}(\psi_2 - y_2)^{\beta_2 - 1}}{(\psi_2 - x_2)^{\alpha_2 + \beta_2 + 1}\, \mathrm{B}(\alpha_1, \beta_1)} \frac{(x_2 - \phi_1)^{\alpha_1 - 1}(\psi_1 - x_2)^{\beta_1 - 1}}{(\psi_1 - \phi_1)^{\alpha_1 + \beta_1 + 1}\, \mathrm{B}(\alpha_2, \beta_2)}\, dx_2. \end{aligned}$$

Just as with Example A.2, the result is a piecewise function, since the domain of A will differ depending on if $y_2 < \psi_1$ or $\psi_1 \leq y_2$.

The marginal will therefore be

$$\begin{aligned} f_{Y_2}(y_2) &= \int_A f_{Y_2|X_2}(y_2|x_2) f_{X_2}(x_2)\, dx_2 \\ &= \int_A \frac{(y_2 - x_2)^{\alpha_2 - 1}(\psi_2 - y_2)^{\beta_2 - 1}}{(\psi_2 - x_2)^{\alpha_2 + \beta_2 + 1}\, \mathrm{B}(\alpha_1, \beta_1)} \frac{(x_2 - \phi_1)^{\alpha_1 - 1}(\psi_1 - x_2)^{\beta_1 - 1}}{(\psi_1 - \phi_1)^{\alpha_1 + \beta_1 + 1}\, \mathrm{B}(\alpha_2, \beta_2)}\, dx_2 \\ &= \frac{(\psi_2 - y_2)}{(\psi_1 - \phi_1)^3\, \mathrm{B}(2,2)^2} \int_A \frac{(y_2 - x_2)(x_2 - \phi_1)(\psi_1 - x_2)}{(\psi_2 - x_2)^3}\, dx_2. \end{aligned}$$

Let $C = (\psi_2 - y_2)/[(\psi_1 - \phi_1)^3\, \mathrm{B}(2,2)^2]$. Expanding the polynomial, let

$$a_3 = 1, \qquad a_2 = -\phi_1 - \psi_1 - y_2, \qquad a_1 = \phi_1\psi_1 + \psi_1 y_2 + \phi_1 y_2, \qquad a_0 = -\phi_1\psi_1 y_2$$

so that the marginal becomes

$$f_{Y_2}(y_2) = C \int_A \frac{a_3 x_2^3}{(\psi_2 - x_2)^3} + \frac{a_2 x_2^2}{(\psi_2 - x_2)^3} + \frac{a_1 x_2}{(\psi_2 - x_2)^3} + \frac{a_0}{(\psi_2 - x_2)^3}\, dx_2$$

which can be solved per standard rules of integration:

$$\begin{aligned} f_{Y_2}(y_2) = C \Bigg\{ & a_3 \left[\frac{\psi_2^2(6x_2 - 5\psi_2)}{2(\psi_2 - x_2)^2} - 3\psi_2 \log(\psi_2 - x_2) - x_2 \right] \\ & + a_2 \left[\frac{2\psi_2}{\psi_2 - x_2} - \frac{\psi_2^2}{2(\psi_2 - x_2)^2} - \log(\psi_2 - x_2) \right] + a_1 \left[\frac{2x_2 - \psi_2}{2(\psi_2 - x_2)^2} \right] + a_0 \left[\frac{1}{2(\psi_2 - x_2)^2} \right] \Bigg\} \Bigg|_A. \end{aligned}$$

To obtain a more concise representation, let

$$b_1 = 6\psi_2^2 + 4a_2\psi_2 + 2a_1, \qquad b_0 = -5\psi_2^3 - 3a_2\psi_2^2 - a_1\psi_2 + a_0$$

so that

$$f_{Y_2}(y_2) = C \left[\frac{b_1 x_2 + b_0}{2(\psi_2 - x_2)^2} - (3\psi_2 + a_2) \log(\psi_2 - x_2) - x_2 \right] \Bigg|_A.$$

The region A to integrate over for x_2 will depend on the relationship of y_2 and ψ_1. If $y_2 < \psi_1$, then $A = [\phi_1, y_2]$. If $\psi_1 \leq y_2$, then $A = [\phi_1, \psi_1]$. The final form of the marginal p.d.f. will thus be a piecewise function, for y_2 on the support of $[\phi_1, \psi_2]$:

$$f_{Y_2}(y_2) = \begin{cases} C \left[\dfrac{b_1 y_2 + b_0}{2(\psi_2 - y_2)^2} - \dfrac{b_1\phi_1 + b_0}{2(\psi_2 - \phi_1)^2} - (3\psi_2 + a_2) \log\left(\dfrac{\psi_2 - \phi_1}{\psi_2 - y_2} \right) - y_2 + \phi_1 \right] & \text{for } y_2 < \psi_1, \\ C \left[\dfrac{b_1\psi_1 + b_0}{2(\psi_2 - \psi_1)^2} - \dfrac{b_1\phi_1 + b_0}{2(\psi_2 - \phi_1)^2} - (3\psi_2 + a_2) \log\left(\dfrac{\psi_2 - \phi_1}{\psi_2 - \psi_1} \right) - \psi_1 + \phi_1 \right] & \text{for } \psi_1 \leq y_2. \end{cases}$$

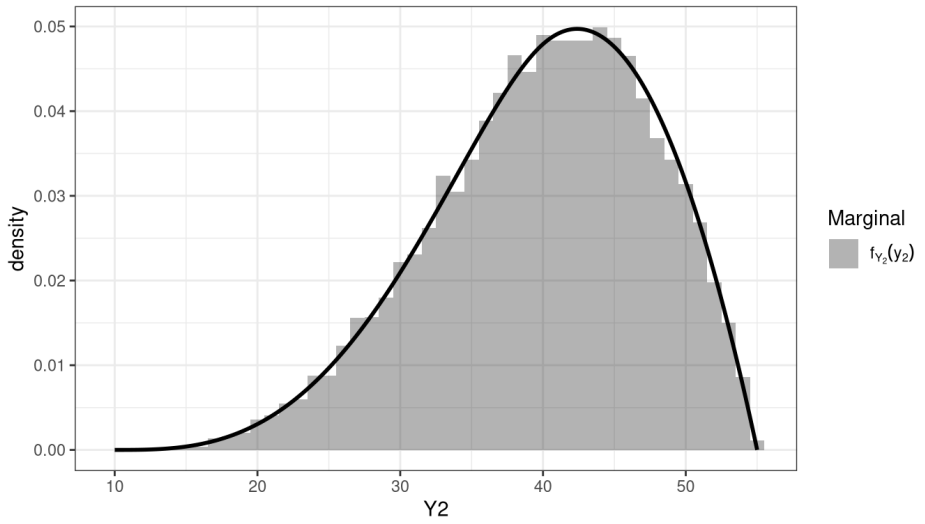


Figure A1: The marginal p.d.f. of Y_2 according to the situation in Example A.4, with samples in grey. This result occurs for $\phi_1 = 10, \psi_1 = 40, \psi_2 = 55$.

Figure A1 illustrates the plot of the marginal p.d.f. of Y_2 obtained analytically along with the results obtained via simulation.

If one were to solve for the other variates, one would change the values of the Beta distribution in the joint density accordingly. E.g., for Y_1, one would use the values of $\alpha_2 = 1, \beta_2 = 3$. Or, if one were to increase the number of the events bounded by the fixed constants, and/or change their conditional structure, the number of terms in the joint probability will change, even as it remains a product of Beta distributions. This of course depends on the use of single-point constants as the ultimate demarcators of *t.p.q.* and *t.a.q.*; any constraint that is itself a density that has been estimated by simulation will warrant simulation for marginalizing any dependent variates.

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